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I. THEOREM CONCERNING THE CONCURRENCY OF THREE LINES.

By E. L. REES, University of Kentucky.

The theorem stating the condition for the collinearity of three points given by their position vectors, is well known to students of vector analysis.[1] It is the purpose of this note to state and prove a similar theorem concerning the concurrency of three lines and to illustrate how this theorem may be used.

THEOREM. *The necessary and sufficient condition that the lines AA′, BB′, and CC′ be concurrent is that the position vectors of the points determining the lines satisfy equations of the form*

$$a\boldsymbol{A} + a'\boldsymbol{A}' = b\boldsymbol{B} + b'\boldsymbol{B}' = c\boldsymbol{C} + c'\boldsymbol{C}'$$

the scalar coefficients of which satisfy the relations

$$a + a' = b + b' = c + c'.^2$$

Proof. It is a necessary condition; for if the lines are concurrent the position vector of the point of intersection may be expressed in terms of each pair of vectors as follows:

$$\boldsymbol{P} = \frac{a_1\boldsymbol{A} + a_1'\boldsymbol{A}'}{a_1 + a_1'} = \frac{b_1\boldsymbol{B} + b_1'\boldsymbol{B}'}{b_1 + b_1'} = \frac{c_1\boldsymbol{C} + c_1'\boldsymbol{C}'}{c_1 + c_1'}$$

and since the sum of the scalar coefficients of each member is unity the condition is satisfied. The reader can easily see that the condition is also satisfied when the lines are parallel or coincident.

It is a sufficient condition; for if we divide member by member we have

$$\frac{a\boldsymbol{A} + a'\boldsymbol{A}'}{a + a'} = \frac{b\boldsymbol{B} + b'\boldsymbol{B}'}{b + b'} = \frac{c\boldsymbol{C} + c'\boldsymbol{C}'}{c + c'},$$

each member of which is a vector terminating in one of the lines, and since these vectors are equal they must terminate in the same point and the lines are therefore concurrent. It is assumed in the proof that the members of the second set of equations are not zero. If they are zero the lines are parallel or coincident.

Very brief and simple proofs of many of the theorems of geometry involving the concurrency of three lines result from the application of this theorem. As illustrations we prove the following.

DESARGUES'S THEOREM. *If two triangles are so situated that the lines joining corresponding vertices are concurrent the corresponding sides intersect in collinear points.*

Proof. If we denote the two triangles by *ABC* and *A′B′C′* then by the above theorem we have for hypothesis

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[1] Cf. Gibbs-Wilson, *Vector Analysis*, New York, 1909, p. 31.—EDITOR.

[2] We shall understand that the term concurrent as used here includes the special cases for which the lines are parallel or coincident.

$$\begin{cases} a\boldsymbol{A} + a'\boldsymbol{A}' = b\boldsymbol{B} + b'\boldsymbol{B}' = c\boldsymbol{C} + c'\boldsymbol{C}', \\ \quad a + a' \quad = \quad b + b' \quad = \quad c + c'. \end{cases}$$

Calling the intersections of AB, BC, and CA with the corresponding sides of the other triangle, D, E, and F respectively, and proceeding in the usual way we find[1]

$$(a - b)\boldsymbol{D} = a\boldsymbol{A} - b\boldsymbol{B},$$

$$(b - c)\boldsymbol{E} = b\boldsymbol{B} - c\boldsymbol{C},$$

$$(c - a)\boldsymbol{F} = c\boldsymbol{C} - a\boldsymbol{A}.$$

Adding we have

$$(a - b)\boldsymbol{D} + (b - c)\boldsymbol{E} + (c - a)\boldsymbol{F} = 0.$$

Since the sum of the scalar coefficients of this equation is zero the theorem is proved.

It will be noted that the triangles may be in the same or different planes. Thus the proof is perfectly general.

CEVA'S THEOREM. *If in the triangle ABC the points A', B', and C' are taken on the sides BC, CA, and AB respectively, in such a way that*

$$(AC'/C'B) \cdot (BA'/A'C) \cdot (CB'/B'A) = 1,$$

then the lines AA', BB', and CC' are concurrent.

Proof. Calling the first and second ratios s/r and t/s respectively, the third ratio must be r/t by the hypothesis of the theorem. We have then

$$(s + t)\boldsymbol{A}' = s\boldsymbol{B} + t\boldsymbol{C},$$

$$(t + r)\boldsymbol{B}' = t\boldsymbol{C} + r\boldsymbol{A},$$

$$(r + s)\boldsymbol{C}' = r\boldsymbol{A} + s\boldsymbol{B}.$$

Adding $r\boldsymbol{A}$ to the first equation, $s\boldsymbol{B}$ to the second, and $t\boldsymbol{C}$ to the third, we see that the second members of the resulting equations are equal. Hence

$$(s + t)\boldsymbol{A}' + r\boldsymbol{A} = (t + r)\boldsymbol{B}' + s\boldsymbol{B} = (r + s)\boldsymbol{C}' + t\boldsymbol{C}.$$

The scalar coefficients of these equations satisfy the condition of the theorem, hence the proof is complete.

II. ON A THEOREM IN THE THEORY OF PROBABILITIES.

BY CHARLOTTE DICKSON, American Telephone and Telegraph Co., New York.

In this MONTHLY for October, 1919, Mr. Bancroft H. Brown gives a method for determining the "average number of rolls needed to decide a game of craps."

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[1] We assume here that $a - b$, $b - c$, and $c - a$ are not zero. If any of these differences vanish then some of the corresponding sides are parallel, *i.e.*, intersect at infinity. Our justification for extending the proof to include the points at infinity lies in the fact that the points of intersection remain collinear when one or more of them recede to infinity.